

A Surface Model of Slothy Joe

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Abstract

We built a three-dimensional model of Slothy Joe, a plush sloth, using ellipsoids, spheres, and cylindrical surfaces in Desmos. This paper describes him, lists the equations and restrictions used for each part of his body, explains how the parameters control the shape of each surface, and reflects on where the model matches the real Slothy Joe and where it does not.

1 Object Description

We chose to model Slothy Joe, the mascot of the Treasure Valley Math and Science Center (TVMSC). Slothy Joe is a sloth plush with golden eyes, and he is dressed in TVMSC merchandise and proudly wears a TVMSC flag. We found him in the display case on the second floor of TVMSC.

We picked him because he's iconic, but also because most of his body can be broken into ellipsoids and cylinders, which makes him a good fit for the surfaces we studied in this unit. He also presents a few less obvious problems. His head has several distinct color regions, and each one has to be modeled by restricting the coordinates on a single surface. His eyes also point in different directions, so they cannot share one set of bounds. Figure 1 shows Slothy Joe in his display case.



Figure 1: Slothy Joe in the display case on the second floor of TVMSC.

2 Mathematical Analysis

2.1 Coordinate System and Scale

We built the model in Desmos 3D. We started with his head only and centered it at the origin. When we extended the model to his whole body, we kept the center of his head at the origin and placed every other part relative to it. The scale is 1 cm on the real Slothy Joe to 2 units in the model, so a length of L centimeters becomes $2L$ units. A semi-axis of a units in an equation therefore corresponds to a real length of $a/2$ centimeters.

2.2 Surfaces Used

Every part of the model is one of the following surfaces, with center (h, k, l) .

An ellipsoid with semi-axes a , b , and c :

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} + \frac{(z-l)^2}{c^2} = 1.$$

A sphere is the special case $a = b = c = r$.

A cylindrical surface with an elliptical cross section, running along the z -axis between $z = z_1$ and $z = z_2$:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad z_1 \leq z \leq z_2.$$

Cylinders along the x - or y -axis use the same form just with the variables permuted.

2.3 Model Equations

Table 1 lists each part of the model with its surface type and parameters. The values in the table are in model units, so divide by 2 to get centimeters.

The full list of Desmos entries is given below.

Colors

$$G = \text{rgb}(128, 128, 128)$$

$$W = \text{rgb}(255, 255, 255)$$

$$G_1 = \text{rgb}(255, 215, 0)$$

$$B = \text{rgb}(0, 0, 0)$$

$$S = \text{rgb}(245, 243, 191)$$

$$B_R = \text{rgb}(101, 67, 33)$$

Right Iris

$$(x-7)^2 + (y+5)^2 + (z+0.8)^2 = 25$$

Right Pupil

$$(x-7)^2 + (y+5)^2 + (z+0.8)^2 = 25.01 \{5x - 2y > 68\}$$

Left Iris

$$(x-7)^2 + (y-5)^2 + (z+0.8)^2 = 25$$

Left Pupil

$$(x-7)^2 + (y-5)^2 + (z+0.8)^2 = 25.01 \{5x + 2y > 68\}$$

Table 1: Parts of the model with their surface types, centers, and parameters.

Part	Surface	Center (h, k, l)	Semi-axes or radius, bounds
Right Iris	Sphere	$(7, -5, -0.8)$	5
Right Pupil	Sphere, restricted	$(7, -5, -0.8)$	$\sqrt{25.01}, \{5x - 2y > 68\}$
Left Iris	Sphere	$(7, 5, -0.8)$	5
Left Pupil	Sphere, restricted	$(7, 5, -0.8)$	$\sqrt{25.01}, \{5x + 2y > 68\}$
Main Head	Ellipsoid	$(0, 0, 0)$	12, 14, 12
Right Mask	Ellipsoid, restricted	$(0, 0, 0)$	12, 14, 12, $\{10x - 2y - 2z > 80\}$
Left Mask	Ellipsoid, restricted	$(0, 0, 0)$	12, 14, 12, $\{10x + 2y - 2z > 80\}$
Right Inner Band	Ellipsoid, restricted	$(0, 0, 0)$	12, 14, 12, $\{20x - 10y - 3z > 258\}$
Right Outer Band	Ellipsoid, restricted	$(0, 0, 0)$	12, 14, 12, $\{10x - 2y - 2z > 80, \dots\}$
Left Inner Band	Ellipsoid, restricted	$(0, 0, 0)$	12, 14, 12, $\{20x + 10y - 3z > 258\}$
Left Outer Band	Ellipsoid, restricted	$(0, 0, 0)$	12, 14, 12, $\{10x + 2y - 2z > 80, \dots\}$
Muzzle	Ellipsoid	$(10, 0, -6)$	3, 5, 3.5
Nose	Ellipsoid	$(12.5, 0, -5.8)$	0.7, 1.7, 1.5
Left Sleeve	Cylinder, rotated	$(0, 0, -6)$	3.5, $\{0 > z > -20, 16 > y > 0, \dots\}$
Left Upper Arm	Cylinder, rotated	$(0, 0, -6)$	3.5, $\{0 > z > -20, 16 > y > 0, \dots\}$
Left Elbow	Sphere	$(-1, 13, -19.5)$	4
Left Forearm	Cylinder, restricted	$(x, 13, -19.5)$	4, $\{-2 < x < 16\}$
Left Hand	Sphere	$(16, 13, -19.5)$	4
Right Sleeve	Cylinder, rotated	$(0, 0, -6)$	3.5, $\{0 > z > -20, -16 < y < 0, \dots\}$
Right Upper Arm	Cylinder, rotated	$(0, 0, -6)$	3.5, $\{0 > z > -20, -16 < y < 0, \dots\}$
Right Elbow	Cylinder, restricted	$(x, -13, -19.5)$	4, $\{-2 < x < 16\}$
Right Forearm	Sphere	$(-1, -13, -19.5)$	4
Right Hand	Sphere	$(16, -13, -19.5)$	4
Neck	Elliptical Cyl., restricted	$(-2, 0, z)$	8.5, 8, $\{-6 > z > -9\}$
Lower Back	Elliptical Cyl., restricted	$(-2, 0, z)$	8.5, 8, $\{-21 > z > -24\}$
Butt	Ellipsoid	$(-2, 0, -23)$	8, $\sqrt{70}, \sqrt{55}$
Right Leg	Cylinder, restricted	$(x, -3.9, -25)$	3.5, $\{0 < x < 17\}$
Right Foot	Sphere	$(17, -3.9, -25)$	3.5
Left Leg	Cylinder, restricted	$(x, 3.9, -25)$	3.5, $\{0 < x < 17\}$
Left Foot	Sphere	$(17, 3.9, -25)$	3.5
Shirt	Elliptical Cyl., restricted	$(-2, 0, z)$	8.5, 8, $\{-9 > z > -21\}$
Button	Ellipsoid	$(7, -2.5, -14)$	$\sqrt{0.5}, \sqrt{8}, \sqrt{8}$

Main Head

$$\frac{(x^2)}{36} + \frac{(y^2)}{49} + \frac{(z^2)}{36} = 4$$

Right Mask

$$\frac{(x^2)}{12^2} + \frac{(y^2)}{14^2} + \frac{(z^2)}{12^2} = 1 \{10x - 2y - 2z > 80\}$$

Left Mask

$$\frac{(x^2)}{12^2} + \frac{(y^2)}{14^2} + \frac{(z^2)}{12^2} = 1 \{10x + 2y - 2z > 80\}$$

Right Inner Band

$$\frac{(x^2)}{12^2} + \frac{(y^2)}{14^2} + \frac{(z^2)}{12^2} = 1 \{20x - 10y - 3z > 258\}$$

Right Outer Band

$$\frac{(x^2)}{12^2} + \frac{(y^2)}{14^2} + \frac{(z^2)}{12^2} = 1 \{10x - 2y - 2z > 80 \text{ and } 2z - y < 14.5 \text{ and } 2z + y > -19.5 \text{ and } y < -5.6 \text{ and } -5.5 < z < 3.1\}$$

Left Inner Band

$$\frac{(x^2)}{12^2} + \frac{(y^2)}{14^2} + \frac{(z^2)}{12^2} = 1 \{20x + 10y - 3z > 258\}$$

Left Outer Band

$$\frac{(x^2)}{12^2} + \frac{(y^2)}{14^2} + \frac{(z^2)}{12^2} = 1 \{10x + 2y - 2z > 80 \text{ and } 2z + y < 14.5 \text{ and } 2z - y > -19.5 \text{ and } y > 5.6 \text{ and } -5.5 < z < 3.1\}$$

Muzzle

$$\left(\frac{y^2}{5^2}\right) + \frac{(x-10)^2}{3^2} + \frac{(z+6)^2}{3.5^2} = 1$$

Nose

$$\frac{(x-12.5)^2}{0.7^2} + \frac{y^2}{1.7^2} + \frac{(z+5.8)^2}{1.5^2} = 1$$

Left Sleeve

$$x^2 + y^2 \cos^2\left(\frac{\pi}{4}\right) + 2y(z+6) \sin\left(\frac{\pi}{4}\right) \cos\left(\frac{\pi}{4}\right) + (z+6)^2 \sin^2\left(\frac{\pi}{4}\right) = (3.5)^2$$

$$\{0 > z > -20 \text{ and } 16 > y > 0 \text{ and } z > y - 25\}$$

Left Upper Arm

$$x^2 + y^2 \cos^2 \left(\frac{\pi}{4} \right) + 2y(z+6) \sin \left(\frac{\pi}{4} \right) \cos \left(\frac{\pi}{4} \right) + (z+6)^2 \sin^2 \left(\frac{\pi}{4} \right) = (3.5)^2$$

$$\{0 > z > -20 \text{ and } 16 > y > 0 \text{ and } z < y - 25\}$$

Left Elbow

$$(x+1)^2 + (y-13)^2 + (z+19.5)^2 = 16$$

Left Forearm

$$(y-13)^2 + (z+19.5)^2 = 4^2 \{-2 < x < 16\}$$

Left Hand

$$(x-16)^2 + (y-13)^2 + (z+19.5)^2 = 16$$

Right Sleeve

$$x^2 + y^2 \cos^2 \left(-\frac{\pi}{4} \right) + 2y(z+6) \sin \left(-\frac{\pi}{4} \right) \cos \left(-\frac{\pi}{4} \right) + (z+6)^2 \sin^2 \left(-\frac{\pi}{4} \right) = (3.5)^2$$

$$\{0 > z > -20 \text{ and } -16 < y < 0 \text{ and } z > -y - 25\}$$

Right Upper Arm

$$x^2 + y^2 \cos^2 \left(-\frac{\pi}{4} \right) + 2y(z+6) \sin \left(-\frac{\pi}{4} \right) \cos \left(-\frac{\pi}{4} \right) + (z+6)^2 \sin^2 \left(-\frac{\pi}{4} \right) = (3.5)^2$$

$$\{0 > z > -20 \text{ and } -16 < y < 0 \text{ and } z < -y - 25\}$$

Right Elbow

$$(x+1)^2 + (y+13)^2 + (z+19.5)^2 = 16$$

Right Forearm

$$(y+13)^2 + (z+19.5)^2 = 4^2 \{-2 < x < 16\}$$

Right Hand

$$(x-16)^2 + (y+13)^2 + (z+19.5)^2 = 16$$

Neck

$$\frac{(x+2)^2}{8.5^2} + \frac{y^2}{64} = 1 \{-6 > z > -9\}$$

Lower Back

$$\frac{(x+2)^2}{8.5^2} + \frac{y^2}{64} = 1 \{-21 > z > -24\}$$

Butt

$$\frac{(x+2)^2}{64} + \frac{(y^2)}{70} + \frac{(z+23)^2}{55} = 1$$

Right Leg

$$(y+3.9)^2 + (z+25)^2 = 3.5^2 \{0 < x < 17\}$$

Right Foot

$$(x-17)^2 + (y+3.9)^2 + (z+25)^2 = 3.5^2$$

Left Leg

$$(y-3.9)^2 + (z+25)^2 = 3.5^2 \{0 < x < 17\}$$

Left Foot

$$(x-17)^2 + (y-3.9)^2 + (z+25)^2 = 3.5^2$$

Shirt

$$\frac{(x+2)^2}{8.5^2} + \frac{y^2}{64} = 1 \{-9 > z > -21\}$$

Button

$$\frac{(x-7)^2}{0.5} + \frac{(y+2.5)^2}{8} + \frac{(z+14)^2}{8} = 1$$

2.4 Restrictions and Color Regions

Some regions of his head are a different color from the rest of the surface. We modeled these by keeping the same ellipsoid equation and adding inequalities on x , y , and z so the equation only draws the part of the surface that has that color. Each region is a separate entry with its own bounds.

The pupils needed more care. His two eyes sit on different faces of his head, so the pupils point in different directions. We used a bird's-eye view of Slothy Joe to find the direction each pupil faces. For each pupil we then wrote a cutting plane in terms of x and y , of the form

$$px + qy \geq s,$$

and kept only the part of the sphere on the outward side of that plane. The constants p , q , and s are chosen so the plane is perpendicular to the direction the pupil faces. Their values are $(7, 5, -0.8)$ for the left pupil and $(7, -5, -0.8)$ for the right pupil.

2.5 Effect of the Parameters

The center (h, k, l) moves a surface without changing its shape. In an ellipsoid, the semi-axes a , b , and c set the extent of the surface along the x -, y -, and z -directions. Increasing one semi-axis stretches the ellipsoid along that axis only. If two semi-axes are equal, the ellipsoid is a surface of revolution about the third axis, and if all three are equal the surface is a sphere. We used this to make his head slightly wider than it is tall, and to make his torso longer along the vertical axis than in the other two directions.

For a cylinder, a and b set the size of the elliptical cross section. Setting $a = b$ gives a circular cross section, and the bounds z_1 and z_2 set the length. Changing a or b changes the thickness of a limb without changing its length, and changing the bounds does the opposite. His limbs were the easiest parts to adjust for this reason, since thickness and length could be matched to the real measurements separately.

3 Documentation

3.1 Measurements

Figure 2 shows Slothy Joe with the measurements we used. All lengths are in centimeters and were measured directly on him.



Figure 2: Labeled diagram of Slothy Joe showing the measurements used in the model.

3.2 Reference Photograph and Model Render

Figure 3 is the photograph we used as a reference while building the model, and Figure 4 is the finished Desmos render. Use the following link to view the full Desmos graph:

<https://www.desmos.com/3d/nolgdweoux>.



Figure 3: Reference photograph of Slothy Joe.

3.3 Simplifications and Assumptions

To keep the model manageable in Desmos, we made a few simplifications.

First, we assumed that all of Slothy Joe's rounded shapes can be modeled by ellipsoids. This covers his head, torso, eyes, and nose, with the pupils treated as spheres. The real plush is soft and a little lumpy, so an ellipsoid captures the overall size and shape of each part but not the small bumps and creases.

Second, we assumed that his limbs can be modeled by cylinders. A cylinder has the same cross section along its whole length, while his arms and legs bend and change thickness slightly, so this is a good first approximation and not an exact match.

Third, we left out the text and diagrams on his shirt and merchandise. Modeling lettering and graphics would need something like a text engine in Desmos, along with a large number of complicated equations for every curve. That would add a lot of work without changing the 3D shapes we were studying, so we modeled only the shapes and left the printed details out.

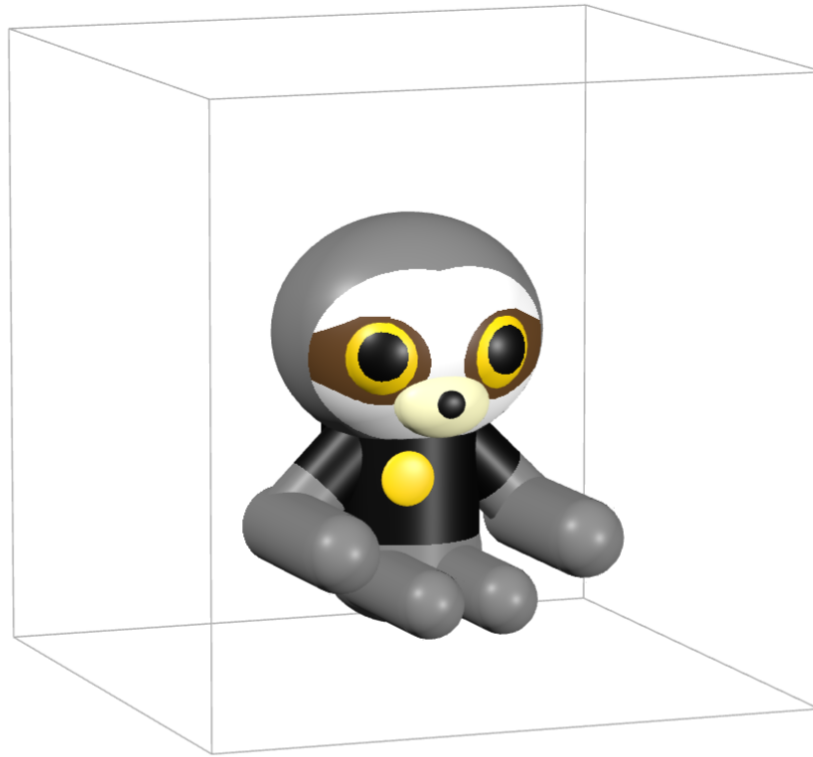


Figure 4: Desmos render of the finished model.

4 Reflection

The main challenge in this project was setting the expression bounds. The clearest example was the pupils. Since his two eyes are not on the same face of his head, the pupils cannot point the same way, and a single set of bounds would have drawn one of them in the wrong place. We solved this by studying the bird's-eye view of Slothy Joe to find each pupil's direction and then cutting each pupil with its own plane in x and y , as described in Section 2.4.

The model matches the real Slothy Joe well in its overall measurements. It is less accurate in the fine shape of his head and legs, which are lumpy and uneven in real life. spheres and cylinders are smooth and symmetric, so they cannot reproduce that. Elliptic cones would also fall short for the same reason.

As described in Section 3.3, we assumed that Slothy Joe's body is made entirely of ellipsoids and cylinders. This is a reasonable assumption because he is mostly smooth and symmetric, and the parts that stick out, such as his limbs, are close to cylindrical. If we continued the project, the next step would be to use surfaces with varying cross sections to capture the shape of his legs and head more closely.